Dispersion

A wave, travelling through a medium, can be a "pure" wave with a single wavenumber k. Or it can also be a result of a superposition of a few waves with different k's.

Recall $k = \frac{2\pi}{\lambda}$ ← distance related, $\omega = \frac{2\pi}{T}$ ← time related.

Remember the relation between the angular frequency of a wave and its wave number, covered in a wave review.

$$v_p = \frac{\omega}{k}$$

Looking at this relation, it's is relatively easy to make a few observations.

(1) If the waves with different wave numbers k, making up another wave, are travelling at the same speed then they would have different frequencies ω, such that ω/k ratio would be constant.

Therefore, the resulting travelling wave shape, would not change.

(2) On the other hand, if $\frac{\omega}{k}$ ratio is not constant for every consituent wave, then it means that every component of that superposed wave moves with its own speed v.

This is what we call **dispersion**. It is when the waves with different wave numbers k propagate at different speeds v.

Rewriting the $\frac{\omega}{k}$ relation above gives:

$$\omega = v_p k$$

← dispersion relation

This relation is important to describe how waves propagate in a particular medium.

Note: v_p here is called "phase velocity". It is a velocity with which any frequency component of the wave travels.

There is another velocity involved called "group velocity". It is the velocity, with the overall shape (also called envelope), of the wave travels.

$$v_g = \frac{\partial \omega}{\partial k}$$

→ if the $\frac{\partial \omega}{\partial k} = 0$ → no dispersion and all wave components travel at the same speed.